

Sequences and Series

Algebra 2
Chapter 12

Algebra II 12

- ◆ This Slideshow was developed to accompany the textbook
 - ◆ *Larson Algebra 2*
 - ◆ *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
 - ◆ *2011 Holt McDougal*
- ◆ Some examples and diagrams are taken from the textbook.

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12.1 Define and Use Sequences and Series

◆ Sequence

- ◆ Function whose domain are integers
- ◆ List of numbers that follow a rule

◆ 2, 4, 6, 8, 10

◆ Finite

◆ 2, 4, 6, 8, 10, ...

◆ Infinite

n is like x , a_n is like y

12.1 Define and Use Sequences and Series

- ◇ Rule

- ◇ $a_n = 2n$

- ◇ Domain: (n)

- ◇ Term's location (1st, 2nd, 3rd...)

- ◇ Range: (a_n)

- ◇ Term's value (2, 4, 6, 8...)

12.1 Define and Use Sequences and Series

◆ Writing rules for sequences

◆ Look for patterns

◆ Guess-and-check

◆ $\frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \frac{2}{625}, \dots$

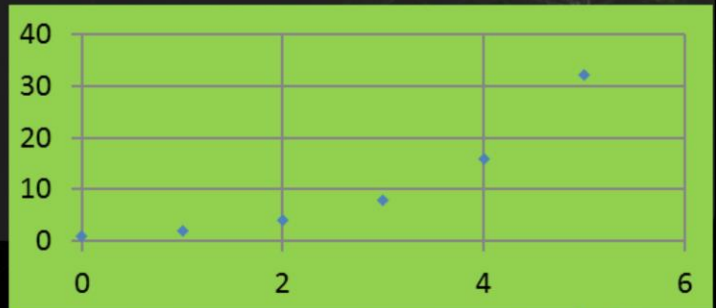
◆ $3, 5, 7, 9, \dots$

$$2/5^1, 2/5^2, 2/5^3, 2/5^4, \dots \rightarrow a_n = 2/5^n$$

$$2(1)+1, 2(2)+1, 2(3)+1, \dots \rightarrow a_n = 2n + 1$$

12.1 Define and Use Sequences and Series

- ◆ To graph
 - ◆ n is like x ; a_n is like y
 - ◆ The graph will be dots
 - ◆ Do NOT connect the dots



The n 's are integers so there is no values between the integers.

12.1 Define and Use Sequences and Series

◆ Series

◆ Sum of a sequence

◆ $2, 4, 6, 8, \dots \rightarrow$ sequence

◆ $2 + 4 + 6 + 8 + \dots \rightarrow$ series

12.1 Define and Use Sequences and Series

◆ Sigma notation

◆ Finite

$$2 + 4 + 6 + 8 = \sum_{i=1}^4 2i$$

Upper limit

Lower limit

Index of summation
(variable)

◆ Infinite

$$2 + 4 + 6 + 8 + \cdots = \sum_{i=1}^{\infty} 2i$$

12.1 Define and Use Sequences and Series

◇ Write as a summation

◇ $4 + 8 + 12 + \cdots + 100$

◇ $2 + \frac{3}{4} + \frac{4}{9} + \frac{5}{16} + \cdots$

$a_n = 4n$, lower limit = 1, upper limit = 25

$$\sum_{n=1}^{25} 4n$$

$a_n = (n+1)/n^2$, lower limit = 1, upper limit = ∞

$$\sum_{n=1}^{\infty} \frac{(n+1)}{n^2}$$

Note that the index may be any letter.

12.1 Define and Use Sequences and Series

◆ Find the sum of the series

$$\sum_{k=5}^{10} k^2 + 1$$

$$5^2 + 1 + 6^2 + 1 + 7^2 + 1 + 8^2 + 1 + 9^2 + 1 + 10^2 + 1 = 361$$

12.1 Define and Use Sequences and Series

◆ Some shortcut formulas

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

12.1 Define and Use Sequences and Series

Find the sum of the series

$$\sum_{k=1}^{10} 3k^2 + 2$$

798 #3-63 every other odd, $65 + 3 = 20$

$$3 \frac{n(n+1)(2n+1)}{6} + 2n = 3 \frac{10(10+1)(2(10)+1)}{6} + 2(10) = 1175$$

Quiz

◊ [12.1 Homework Quiz](#)

12.2 Analyze Arithmetic Sequences and Series

◆ Arithmetic Sequences

- ◆ Common difference (d) between successive terms

- ◆ Add the same number each time

- ◆ 3, 6, 9, 12, 15, ...

- ◆ $d = 3$

◆ Is it arithmetic?

- ◆ -10, -6, -2, 0, 2, 6, 10, ...

- ◆ 5, 11, 17, 23, 29, ...

No

Yes, $d = 6$

12.2 Analyze Arithmetic Sequences and Series

◆ Formula for n^{th} term

$$\text{◆ } a_n = a_1 + (n - 1)d$$

◆ Write a rule for the n^{th} term

$$\text{◆ } 32, 47, 62, 77, \dots$$

$$d = 15$$

$$a_n = 32 + (n-1)15 = 32 + 15n - 15 \rightarrow a_n = 17 + 15n$$

12.2 Analyze Arithmetic Sequences and Series

- ◆ One term of an arithmetic sequence is $a_8 = 50$. The common difference is 0.25. Write the rule for the n^{th} term.

$$a_n = a_1 + (n - 1)d$$

$$50 = a_1 + (8 - 1)0.25 \rightarrow 50 = a_1 + 1.75 \rightarrow 48.25 = a_1$$

$$a_n = 48.25 + (n - 1)0.25 \rightarrow a_n = 48.25 + 0.25n - 0.25 \rightarrow a_n = 48 + 0.25n$$

12.2 Analyze Arithmetic Sequences and Series

- ◆ Two terms of an arithmetic sequence are $a_5 = 10$ and $a_{30} = 110$. Write a rule for the n^{th} term.

$$a_n = a_1 + (n - 1)d$$

$$10 = a_1 + (5 - 1)d$$

$$\rightarrow 10 = a_1 + 4d$$

$$110 = a_1 + (30 - 1)d$$

$$\rightarrow 110 = a_1 + 29d$$

Linear combination

$$-10 = -a_1 - 4d$$

$$\underline{110 = a_1 + 29d}$$

$$100 = 25d$$

$$d = 4$$

Substitute

$$10 = a_1 + 4d \rightarrow 10 = a_1 + 4(4) \rightarrow a_1 = -6$$

Rule

$$a_n = -6 + (n - 1)4 \rightarrow a_n = -6 + 4n - 4 \rightarrow a_n = 4n - 10$$

12.2 Analyze Arithmetic Sequences and Series

◆ Sum of a finite arithmetic series

$$◆ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

◆ Rewrite

$$◆ 1 + 2 + 3 + 4 + 5$$

$$◆ \underline{10 + 9 + 8 + 7 + 6}$$

$$◆ 11 + 11 + 11 + 11 + 11 = 5(11) = 55$$

◆ Formula

$$◆ S_n = n \left(\frac{a_1 + a_n}{2} \right)$$

From example:

First and last ($a_1 + a_n$) = 11

10 numbers but only half as many pairs ($n/2$)

12.2 Analyze Arithmetic Sequences and Series

- ◆ Consider the arithmetic series
 - ◆ $20 + 18 + 16 + 14 + \dots$
- ◆ Find the sum of the first 25 terms.

$$a_{25} = 20 + (25-1)(-2) = -28$$

$$S_{25} = 25((20 + -28)/2) = -100$$

12.2 Analyze Arithmetic Sequences and Series

◆ Consider the arithmetic series

$$\text{◆ } 20 + 18 + 16 + 14 + \dots$$

◆ Find n such that $S_n = -760$

◆ 806 #3-63 every other odd, $65 + 3 = 20$

$$a_n = 20 + (n-1)(-2) = 22 - 2n$$

$$S_n = -760 = n((20 + 22 - 2n)/2) \rightarrow -1520 = n(42 - 2n) \rightarrow -1520 = 42n - 2n^2 \rightarrow 2n^2 -$$

$$42n - 1520 = 0 \rightarrow n^2 - 21n - 760 = 0 \rightarrow (n+19)(n-40) = 0 \rightarrow n = 40, -19$$

Quiz

◊ [12.2 Homework Quiz](#)

12.3 Analyze Geometric Sequences and Series

◆ Created by multiplying by a common ratio (r)

◆ Are these geometric sequences?

◆ 1, 2, 6, 24, 120, ...

◆ 81, 27, 9, 3, 1, ...

No

Yes $r = 1/3$

12.3 Analyze Geometric Sequences and Series

◆ Formula for n^{th} term

$$\text{◆ } a_n = a_1 \cdot r^{n-1}$$

◆ Write a rule for the n^{th} term and find a_8 .

$$\text{◆ } 5, 2, 0.8, 0.32, \dots$$

$$r = 2/5$$

$$a_n = 5(2/5)^{n-1}$$

$$a_8 = 5(2/5)^7 = 0.008192$$

12.3 Analyze Geometric Sequences and Series

- ◆ One term of a geometric sequence is $a_4 = 3$ and $r = 3$. Write the rule for the n^{th} term.

$$a_4 = 3 = a_1 3^{4-1} \rightarrow 3 = a_1 27 \rightarrow a_1 = 1/9$$

$$a_n = (1/9) 3^{n-1}$$

12.3 Analyze Geometric Sequences and Series

- ◆ If two terms of a geometric sequence are $a_2 = -4$ and $a_6 = -1024$, write rule for the n^{th} term.

$$a_2 = -4 = a_1 r^{2-1} \rightarrow -4 = a_1 r$$

$$a_6 = -1024 = a_1 r^{6-1} \rightarrow -1024 = a_1 r^5$$

Solve first for a_1 : $a_1 = -4/r$

Plug into second: $-1024 = (-4/r)r^5 \rightarrow -1024 = -4r^5/r \rightarrow -1024 = -4r^4 \rightarrow 256 = r^4 \rightarrow r = 4$

Plug back into first: $a_1 = -4/4 \rightarrow a_1 = -1$

Write rule: $a_n = -1 \cdot 4^{n-1}$

12.3 Analyze Geometric Sequences and Series

◆ Sum of geometric series

$$◆ S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

◆ Find the sum of the first 10 terms of

$$◆ 4 + 2 + 1 + \frac{1}{2} + \dots$$

$$r = \frac{1}{2}, a_1 = 4$$

$$S_{10} = 4((1 - (\frac{1}{2})^{10})/(1 - \frac{1}{2})) = 4(.99902/.5) = 7.992 = 1023/128$$

12.3 Analyze Geometric Sequences and Series

◆ Find n such that $S_n = 31/4$

◆ $4 + 2 + 1 + \frac{1}{2} + \dots$

◆ 814 #3-47 every other odd, 49, 51, 53, 57, 59 + 3 = 20

$$\begin{aligned} 31/4 &= 4((1 - (1/2)^n)/(1 - 1/2)) \rightarrow 31/16 = (1 - (1/2)^n)/(1/2) \rightarrow 31/32 = 1 - (1/2)^n \rightarrow - \\ 1/32 &= -(1/2)^n \rightarrow 1/32 = (1/2)^n \rightarrow \log(1/32) = n \log(1/2) \rightarrow n = (\log(1/32)/\log(1/2)) \\ &= 5 \end{aligned}$$

Quiz

◆ [12.3 Homework Quiz](#)

12.4 Find the Sums of Infinite Geometric Series



Think of the box a 1 whole piece

12.4 Find the Sums of Infinite Geometric Series


$$\frac{1}{2}$$

Cut in half

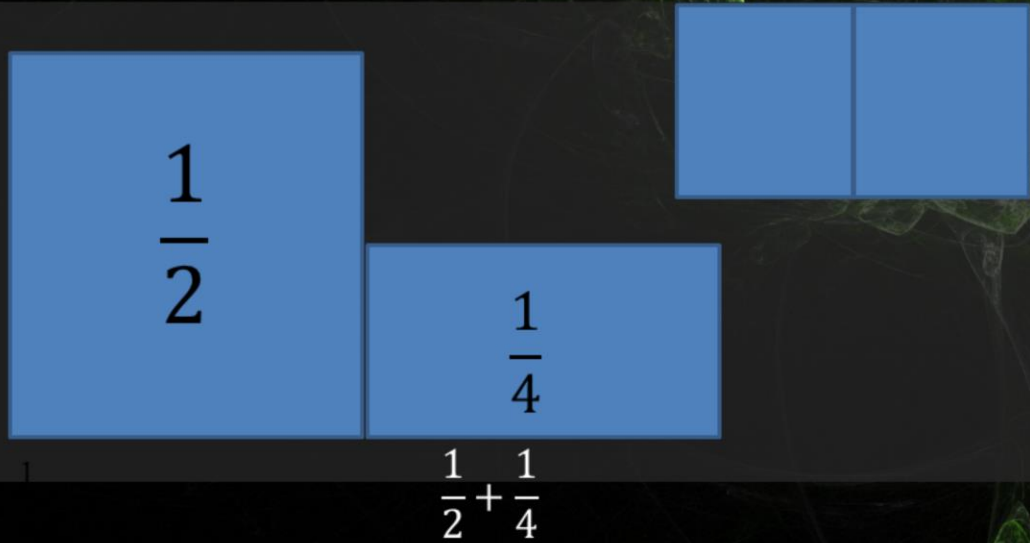
12.4 Find the Sums of Infinite Geometric Series


$$1$$
$$\frac{1}{2}$$


$$\frac{1}{2}$$

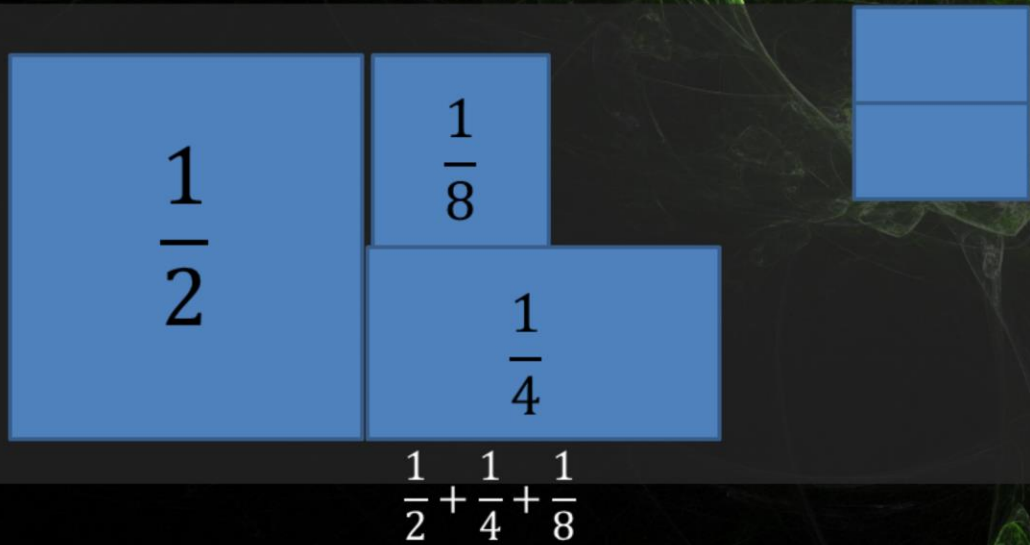
Cut in half

12.4 Find the Sums of Infinite Geometric Series



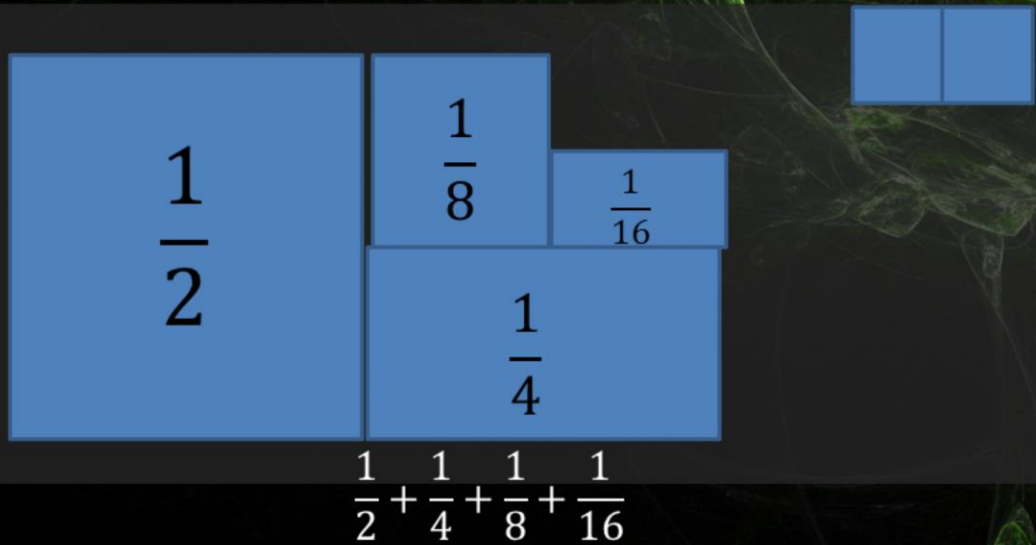
Cut in half

12.4 Find the Sums of Infinite Geometric Series



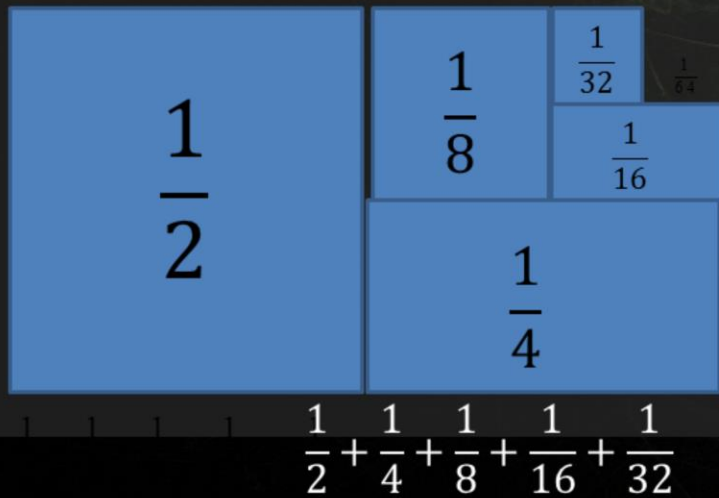
Cut in half

12.4 Find the Sums of Infinite Geometric Series



Cut in half

12.4 Find the Sums of Infinite Geometric Series



What is the sum of the pieces if we keep cutting forever?

What is the sum of the pieces if we keep going? 1 piece

12.4 Find the Sums of Infinite Geometric Series

◆ Sum of an infinite geometric series

$$◆ S = \frac{a_1}{1-r}$$

◆ $|r| < 1$

◆ If $|r| > 1$, then no sum (∞)

12.4 Find the Sums of Infinite Geometric Series

Find the sum

$$\sum_{i=1}^{\infty} 2(0.1)^{i-1}$$

$$12 + 4 + \frac{4}{3} + \frac{4}{9} + \dots$$

$$a_1 = 2(0.1)^{1-1} = 2, r = 0.1$$
$$S = 2/(1-0.1) = 2/.9 = 20/9$$

$$a_1 = 12, r = 1/3$$
$$S = 12/(1-1/3) = 12/(2/3) = 36/2 = 18$$

12.4 Find the Sums of Infinite Geometric Series

- ◆ An infinite geometric series has $a_1 = 5$ has sum of $27/5$. Find the common ratio.

$$27/5 = 5/(1-r) \rightarrow 27(1-r) = 5*5 \rightarrow 1-r = 25/27 \rightarrow -r = -2/27 \rightarrow r = 2/27$$

12.4 Find the Sums of Infinite Geometric Series

◇ Write $0.27272727\dots$ as a fraction.

Write the repeating unit as a sum of fractions

$$27/100 + 27/10000 + 27/1000000 + \dots$$

$$a_1 = 27/100, r = 1/100$$

$$S = (27/100)/(1 - (1/100)) = (27/100)/(99/100) = 27/99 = 3/11$$

12.4 Find the Sums of Infinite Geometric Series

◆ Write $0.41666666\ldots$ as a fraction.

◆ 823 #3-33 odd, 37, 39, $42 + 1 = 20$

$41/100 + 6/1000 + 6/10000 + 6/100000 + \ldots$
Ignore the $41/100$ for now.

$$a_1 = 6/1000, r = 1/10$$
$$S = (6/1000)/(1 - 1/10) = (6/1000)/(9/10) = 60/9000 = 1/150$$

Now add the $41/100$

$$41/100 + 1/150 \rightarrow (41 \cdot 3)/300 + (1 \cdot 2)/300 \rightarrow 123/300 + 2/300 \rightarrow 125/300 \rightarrow 5/12$$

Quiz

◊ [12.4 Homework Quiz](#)

12.5 Use Recursive Rules with Sequences and Functions

◆ Explicit Rule

◆ Gives the n^{th} term directly

◆ $a_n = 2 + 4n$

◆ Recursive Rule

◆ Each term is found by knowing the previous term

◆ $a_1 = 6; a_n = a_{n-1} + 4$

Both these rules give the same sequence

12.5 Use Recursive Rules with Sequences and Functions

◇ Write the first 5 terms

◇ $a_1 = 1, a_n = (a_{n-1})^2 + 1$

◇ $a_1 = 2, a_2 = 2, a_n = a_{n-2} - a_{n-1}$

$$a_1 = 1, a_2 = 1^2 + 1 = 2, a_3 = 2^2 + 1 = 5, a_4 = 5^2 + 1 = 26, a_5 = 26^2 + 1 = 677$$

$$a_1 = 2, a_2 = 2, a_3 = 2 - 2 = 0, a_4 = 2 - 0 = 2, a_5 = 0 - 2 = -2$$

12.5 Use Recursive Rules with Sequences and Functions

◆ Write the rules for the arithmetic sequence where $a_1 = 15$ and $d = 5$.

◆ Explicit

◆ Recursive

Explicit: $a_n = 15 + (n-1)5 \rightarrow a_n = 5n + 10$

Recursive: $a_1 = 15, a_n = a_{n-1} + 5$

12.5 Use Recursive Rules with Sequences and Functions

◆ Write the rule for the geometric sequence where $a_1 = 4$ and $r = 0.2$

◆ Explicit

◆ Recursive

Explicit: $a_n = 4(0.2)^{n-1}$

Recursive: $a_1 = 4, a_n = 0.2a_{n-1}$

12.5 Use Recursive Rules with Sequences and Functions

◆ Write a recursive rule for

◆ 1, 1, 4, 10, 28, 76, ...

◆ 1, 2, 2, 4, 8, 32, ...

$$a_1 = 1, a_2 = 1, a_n = 2(a_{n-2} + a_{n-1})$$

$$a_1 = 1, a_2 = 2, a_n = (a_{n-2})(a_{n-1})$$

12.5 Use Recursive Rules with Sequences and Functions

- ◆ Iterations
 - ◆ Repeated composition of functions
 - ◆ $f(f(f(x)))$
 - ◆ Use x to find $f(x)$
 - ◆ Use that value to find the next $f(x)$
 - ◆ $x_1 = f(x_0)$; $x_2 = f(x_1)$, ...
- ◆ Find the first three iterations of the function.
 - ◆ $f(x) = 4x - 3$, $x_0 = 2$

◆ 830 #3-31 odd, 35-39 odd, 43, $45 + 5 = 25$

$$x_0 = 2$$

$$x_1 = f(2) = 4(2) - 3 = 5$$

$$x_2 = f(5) = 4(5) - 3 = 17$$

$$x_3 = f(17) = 4(17) - 3 = 65$$

Quiz

◊ [12.5 Homework Quiz](#)

12.Review

843 #choose 20 = 20

12 CHAPTER TEST

Tell whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain.

1. 5, 9, 13, 17, ... 2. 3, 6, 12, 24, ... 3. 40, 10, $\frac{5}{2}$, $\frac{5}{8}$, ... 4. 4, 7, 12, 19, ...

Write the first six terms of the sequence.

5. $a_n = 6 + n^2$ 6. $a_n = 7n^3$ 7. $a_1 = 4$
 $a_n = 5a_{n-1}$ 8. $a_1 = -1$
 $a_n = a_{n-1} + 6$

Write the next term of the sequence, and then write a rule for the n th term.

9. 5, 11, 17, 23, ... 10. 3, 15, 75, 375, ... 11. $\frac{5}{9}, \frac{7}{10}, \frac{8}{13}, \frac{9}{20}, \dots$ 12. 1.6, 3.2, 4.8, 6.4, ...

Find the sum of the series.

13. $\sum_{i=1}^{25} i$ 14. $\sum_{i=1}^{25} i^3$ 15. $\sum_{i=1}^{25} (4i - 9)$ 16. $\sum_{i=1}^{25} (2i + 5)$
17. $\sum_{i=1}^5 9(2)^{i-1}$ 18. $\sum_{i=1}^5 12\left(\frac{1}{3}\right)^{i-1}$ 19. $\sum_{i=1}^5 8\left(\frac{3}{4}\right)^{i-1}$ 20. $\sum_{i=1}^5 20\left(\frac{3}{10}\right)^{i-1}$

Write the repeating decimal as a fraction in lowest terms.

21. 0.111... 22. 0.464646... 23. 0.187187187... 24. 0.3252525...

Write a recursive rule for the sequence.

25. 2, 12, 72, 432, ... 26. 3, 10, 17, 24, ... 27. 135, 45, 15, 5, ... 28. 1, -3, 9, -27, ...

Find the first three iterates of the function for the given initial value.

29. $f(x) = 3x - 7$, $x_0 = 4$ 30. $f(x) = 8 - 5x$, $x_0 = 1$ 31. $f(x) = x^2 + 2$, $x_0 = -1$

32. **QUILTS** Use the pattern of checkerboard quilts shown.



$$n = 1, a_n = 1$$



$$n = 2, a_n = 2$$



$$n = 3, a_n = 5$$



$$n = 4, a_n = 8$$

- a. What does n represent for each quilt? What does a_n represent?
b. Make a table that shows n and a_n for $n = 1, 2, 3, 4, 5, 6, 7$, and 8.
c. Use the rule $a_n = \frac{n^2}{2} + \frac{1}{2}(-1)^n$ to find a_n for $n = 1, 2, 3, 4, 5, 6, 7$, and 8. Compare these values with the results in your table. What can you conclude about the sequence defined by this rule?

33. **AUDITIONS** Several rounds of auditions are being held to cast the three main parts in a play. There are 3072 actors at the first round of auditions. In each successive round of auditions, one fourth of the actors from the previous round remain. Find a rule for the number a_n of actors in the n th round of auditions. For what values of n does your rule make sense?